Physics I
ISI B.Math
Midterm Exam : February 27 , 2020

Total Marks: 70
Time : 3 hours

Answer all questions

1. (**Marks** $= \mathbf{3+3+4+4=14}$)

A particle of mass m is moving under the influence of a central force $\mathbf{F}(\mathbf{r}) = F(r)\hat{\mathbf{r}}$.

(a) Show that the trajectory of the particle must lie in a plane.

(b) Show that the particle must obey Kepler's second law, i.e, the radius vector of the particle will sweep out equal areas in equal times.

(c) Show that the force $\mathbf{F}$ is conservative.

(d) Suppose the particle is moving in a circular orbit under the influence of a force $\mathbf{F}(\mathbf{r}) = -\frac{k}{r^2}\hat{\mathbf{r}}$, where $k > 0$.Show that if k suddenly decreases to half its original value, the particle's orbit becomes parabolic. Will it be possible for the particle to move in a circular orbit if k changes sign ? Explain.

2. (**Marks** $= \mathbf{8+6=14}$)

An electrical circuit consists of an inductance L, resistance R and a capacitance C connected in series with a battery of emf $\mathcal{E}$. The charge passing through the circuit at a time t is given by $q(t)$ and the current $I(t) = \frac{dq}{dt}$. The parameters are such that $R = 2\sqrt{\frac{L}{C}}$. $q = q_0$ and $I = 0$ at t =0. Kirchoff's equation around the circuit is given by

$$L\frac{dI}{dt} + RI + \frac{q}{C} = \mathcal{E}$$

a) Solve this equation to find $q(t)$. Exploit the analogy between the electrical system and the mass spring system carefully in order to do this.
b) Now remove the resistance from the circuit and find $q(t)$ with the same initial conditions.

3.(**Marks** $= \mathbf{6+8=14}$)

a) A particle with polar coordinates r, θ which are functions of time t is moving in a plane. The velocity and acceleration of the particle can be written in plane polar coordinates as $\mathbf{v} = v_r\hat{\mathbf{r}} + v_\theta\hat{\theta}$ and $\mathbf{a} = a_r\hat{\mathbf{r}} + a_\theta\hat{\theta}$. Find $v_r, v_\theta, a_r, a_\theta$ in terms of $r, \theta, \dot{r}, \dot{\theta}, \ddot{r}, \ddot{\theta}$.

b) An insect flies on a spiral trajectory such that its polar coordinates at time t are given by

$$r = be^{\Omega t}$$

$$\theta = \Omega t$$

Find the angle between the velocity and acceleration vectors. If the insect were to move along any trajectory in three dimensional space with constant speed, show that its velocity and acceleration vectors must always be perpendicular to each other.

4.(**Marks = 2 + 3 + 3 + 6 = 14**)

(a) A particle moves under the influence of the potential $U(x) = A/x^2 - B/x$ where $A, B > 0$.
(i) Make a rough sketch of the potential as a function of x
(ii) Show that the motion of the particle consists of either(i) periodic oscillation between two turning points or (ii) an unbounded motion with one turning point, depending upon the value of total energy.
iii) Find the frequency of small oscillations about the stable equilibrium point.

(b) Consider a particle of mass m whose motion starts from rest in a constant gravitational field. If a resisting force proportional to the square of the velocity (i.e, kmv^2) is encountered, show that the distance s the particle falls in accelerating from v_0 to v_1 is given by

$$s(v_0 \to v_1) = \frac{1}{2k} \ln \left[\frac{g - kv_0^2}{g - kv_1^2} \right]$$

5.(**Marks = 2 + 4 + 6 + 2 = 14**)

(a) In an experiment, a particle of mass m and energy E is used to bombard a stationary target particle of mass $2m$. After scattering, the particle of mass m is seen to emerge perpendicular to the direction of the original motion in the lab frame. Assume that the collision is elastic.
(i) Sketch how the collision will appear in the centre of mass frame.
(ii) What percent of the original energy of the bombarding particle will be lost after collision ?

(b) Show that the kinetic energy of a system of particles is equal to the sum of the kinetic energy of a particle of mass M (where M is the sum of the masses of all the particles of the system) moving with the velocity of the centre of mass and the kinetic energy of motion of the individual particles relative to the centre of mass. Hence argue that a collision that is elastic in one frame of reference will be elastic in all inertial frames of reference.

Information you may or may not need:

$$\tan\theta_1 = \frac{\sin\psi}{\cos\psi + \gamma}, \theta_2 = \frac{1}{2}(\pi - \psi), \frac{E_2}{E_0} = \frac{4\gamma}{(\gamma+1)^2}\sin^2(\frac{\psi}{2})$$

θ_1 and θ_2 are the scattering angle and recoil angle in the LAB frame respectively. E_2 and E_0 are the LAB frame energies of the target particle after collision and the incident particle before collision respectively. ψ is the scattering angle in the CM (centre of mass) frame, and $\gamma = \frac{m_1}{m_2}$, the mass ratio of the two particles. All formulae refer to elastic collisions.